# Workshop on Verification and Validation of CFD for Offshore Flows

## L. Eça (IST), G.Vaz (MARIN) October 2012

#### 1. Proposed test cases

The first "Workshop on Verification and Validation of CFD for Offshore Flows" [1] included three test cases:

- I) A 3-D manufactured solution for unsteady turbulent flow (including turbulence quantities of eddy-viscosity models).
- II) The flow around a smooth circular cylinder.
- III) The flow around a straked riser.

The goal of the first exercise is to perform Code Verification, i.e. to check that the discretization error tends to zero with grid refinement and (if possible) demonstrate that the observed order of grid convergence matches the "theoretical order" of the code.

The objectives of the other two cases were twofold:

- Check the consistency between error bars obtained from different numerical solutions of the same mathematical model, i.e. perform Solution Verification.
- Apply the V&V 20 Validation procedure [2] to assess the modeling error. This
  means that experiments and simulations require the estimation of their respective
  uncertainties.

This means that the first test case is focused on Code Verification, whereas the remaining test cases include Solution Verification and Validation.

#### 2. Participants/Presentations

The Workshop included 5 sessions:

- 1. Introduction and Code/Solution Verification.
- 2. Solution Verification.
- 3. Solution Verification/Validation.

- 4. Validation/Overall review of results.
- 5. Discussion and future plans.

#### 2.1 Introduction and Code/Solution Verification.

This session included four presentations:

- "Workshop on Verification and Validation of CFD for Offshore Flows", L.Eça (IST) and G.Vaz (MARIN).
- "Verification of ReFRESCO with a URANS Manufactured Solution", <u>L.Eça</u> (IST), G.Vaz (MARIN) and M.Hoekstra (MARIN).
- "Code Verification of an Unsteady RANS Finite Element Solver", A.Hay (EPM), Pelletier D (EPM), S.Etienne (EPM) and A.Garon (EPM).
- "Verification and Validation of VIV-Sim",
   O.Heynes (MMI Engineering) and D.Dolan (MMI Engineering).

#### 2.2 Solution Verification

Two presentations were included in this session followed by a period of discussion.

- "CFD Calculations and Solution Verification for Smooth Fixed Circular Cylinder in the Range of High Reynolds Numbers",
   G. Rosetti (USP), G.Vaz (MARIN) and A.L.C.Fujarra (USP).
- "Numerical Simulation of Flow around a Smooth Circular Cylinder at Re = 1e5 to 1e6 by Using k-epsilon Turbulence Model",
   M.Zhang (SJTU), Z.Lin(SJTU), S.Fu (SJTU), Q.Zhou (SJTU) and X.Zhang (SJTU).

#### 2.3 Solution Verification/Validation

Three presentations were made in the third session of the Workshop.

- "Flow past a circular cylinder: A verification study using approximate error scaling"
   I.Celik (WVU), J.Escobar (WVU), J.Posada-Montoya (WVU) and A.Gutierrez-Amador (WVU).
- "CFD study of the flow over a smooth cylinder and a straked-riser",
   A.Barbagallo (SBM Offshore).

• "Transient analysis of 3D flow around a smooth cylinder and a straked riser with a lattice-Boltzmann method",

M.Mier-Torrecilla (AR), D.M.Holman (NLT), M. Zurita-Gotor (AR) and J.B. Frandsen (AR).

#### 2.4 Validation/Overall review of results

Two presentations were included in this section followed by a first discussion about the submitted results (focused mainly on Code Verification).

- "V&V study on the flow around a straked riser using URANS supplemented by DES simulations",
  - M.Manzke (TUHH) and T.Rung (TUHH).
- "ReFRESCO Results for Riser Test-Case of V&V Workshop", <u>G.Vaz</u> (MARIN) and D.Rijpkema (MARIN).

# 2.5 Discussion and future plans

The last session of the Workshop was dedicated to the comparison of (part of) the submitted data and to discussion of the plans for the next Workshop (see final remarks).

Organization	Flow Solver	Mathematical Model	Test Case
Instituto Superior Técnico (IST)	ReFRESCO	URANS	Manufactured
Maritime Research Institute			Solution
Netherlands (MARIN)			
École Polytechnique de Montreal (EPM)	CADYF	URANS	Manufactured
			Solution
SBM Offshore	STARCCM	URANS	Cylinder
West Virginia University (WVU)	DREAM	Navier-Stokes	Cylinder
West Virginia University (WVU)	FLUENT	Navier-Stokes	Cylinder
Shangai Jiaotong University (SJTU)	FLUENT	URANS	Cylinder
University of São Paulo (USP)/MARIN	ReFRESCO	URANS	Cylinder
MMI Engineering	VIV-Sim	URANS	Cylinder
Abengoa Research SL (AR)	XFlow	Lattice-	Straked Riser
		Boltzmann	
MARIN	ReFRESCO	URANS	Straked Riser
Hamburg University of Technology	FreSCo+	URANS	Straked Riser
(TUHH)			
University of Michigan	OpenFoam	URANS	Cylinder
Krylov Shipbuilding Research Institute	ANSYS	URANS	Cylinder
	CFX		

Table 1: Organizations, flow solvers, mathematical models and test cases of submitted results.

#### 3. Submissions

There were 12 groups submitting results for the Workshop covering all the proposed test cases. The manufactured solution had the smallest number of submissions (2), whereas the largest number of submissions (8) was registered for the flow around the smooth cylinder. Table 1 presents the name of the organizations that submitted results, the name of the flow solver, the mathematical model used and the test cases addressed.

#### 3.1 Manufactured solution

There were only two participants performing Code Verification with the proposed Manufactured Solution. Since the two exercises presented several alternatives to the proposed manufactured solution (not exactly equal for the two groups), it was decided to skip the formal submission of data for this test case.

#### 3.2 Circular cylinder

There were submissions from 7 participants which submitted results at 5 five different Reynolds numbers with four different turbulence models. The submitted results are grouped by Reynolds number and turbulence model in table 2. We have only included in table 2 the submissions that allowed the estimation of the numerical uncertainty, i.e. submissions that included information from more than one grid and time step.

Reynolds	Turbulence Model	Number of
Number		Submissions
$10^{3}$	k-ω SST	2
$10^{3}$	None	2
$10^{4}$	k-ω SST	2
$10^{5}$	k-ω SST	4
10 <sup>5</sup>	None	2
$10^{5}$	k-ε	1
$10^{5}$	Spalart & Allmaras	1
5×10 <sup>5</sup>	k-ω SST	4
5×10 <sup>5</sup>	None	2
5×10 <sup>5</sup>	k-ε	1
5×10 <sup>5</sup>	Spalart & Allmaras	1
10 <sup>6</sup>	k-ε	1

Table 2: Reynolds and turbulence model of the submissions for the flow around a smooth circular cylinder.

The largest number of submissions with the same turbulence model (4) occurred for the SST version of the k- $\omega$  two-equation model [3] for Reynolds numbers of  $10^5$  and  $5\times10^5$ . There were 2 submissions for three different Reynolds numbers ( $10^3$ ,  $10^5$  and  $5\times10^5$ ) that did not use any turbulence model (identified by None in table 2). Unfortunately, only one submission was made for the standard k- $\varepsilon$  two-equation model [4] and the one-equation model of Spalart & Allmaras [5]. This means that for these flow settings and turbulence model it is not possible to check the consistency of the error bars from different numerical solutions (one of the main goals of the present Solution Verification exercise).

#### 3.3 Straked riser

There were 3 different submissions for the calculation of the flow around a straked riser. Two of them were based on the Reynolds-Averaged Navier-Stokes equations supplemented by the SST k- $\omega$  model [3] and one on a Lattice-Boltzmann solver with a "Wall Adapting Eddy-viscosity model" combined with a LES turbulence closure (as described by the participant). One of the presentations also included results from a DES (Detached Eddy-Simulation) approach based on the k- $\omega$  model. However, the results were still preliminary.

#### 4. Uncertainty estimation for submitted data

A procedure based on a power series representation of the discretization error with different contributions of the space and time discretization was applied to the data submitted to the Workshop. The estimated uncertainty relies on a solution extrapolated to cell size zero ( $h_i = 0$ ) and time step zero ( $\tau_i = 0$ ) using a least squares solution of

$$\phi = \phi_o + \alpha_x \left(\frac{h_i}{h_1}\right)^{p_x} + \alpha_t \left(\frac{\tau_i}{\tau_1}\right)^{p_t}$$

for monotonically convergent solutions. Otherwise, the adopted error description is given by

$$\phi = \phi_o + \alpha_{1x} \left( \frac{h_i}{h_1} \right) + \alpha_{1t} \left( \frac{\tau_i}{\tau_1} \right) + \alpha_{2x} \left( \frac{h_i}{h_1} \right)^2 + \alpha_{2t} \left( \frac{\tau_i}{\tau_1} \right)^2.$$

 $\phi$  stands for any of the flow quantities requested for the Workshop;  $\phi_o$  is the estimate of the exact solution;  $h_i$  is the typical cell size of the space discretization( $h_1$  corresponds to the finest grid);  $\tau_i$  is the time step ( $\tau_1$  corresponds to the smallest time step);  $p_x$  is the observed order of convergence of the space discretization;  $p_t$  is the observed order of convergence of the time discretization;  $\alpha_x$ ,  $\alpha_t$ ,  $\alpha_{1x}$ ,  $\alpha_{1x}$ ,  $\alpha_{2x}$  and  $\alpha_{2t}$  are constants. The details of the procedure are beyond the scope of this report. Nevertheless, we exemplify its use with some illustrations of the application to the submitted data.

Figure 1 presents the fits performed to a submission for the flow around the smooth cylinder at  $Re_D = 10^5$  computed with the SST k- $\omega$  eddy-viscosity two-equation model

[3]. The data corresponds to the average  $(C_{Davg})$ , root mean squared  $(C_{Drms})$  and maximum  $(C_{Dmax})$  values of the drag coefficient and to the amplitude of oscillation (  $\Delta C_D = C_{Dmax} - C_{Dmin}$ ). The submission includes 6 calculations using three grid densities (  $h_i/h_1 = 1$ ,  $h_i/h_1 = 1.9$  and  $h_i/h_1 = 3.6$ ) and three time steps  $(\tau_i/\tau_1 = 1, \tau_i/\tau_1 = 2)$  and  $\tau_i/\tau_1 = 4$ . Of the 9 possible combinations, six calculations were performed:  $h_i/h_1 = 1$ ,  $\tau_i/\tau_1 = 1$ ,  $t_i/t_1 = 1$ .

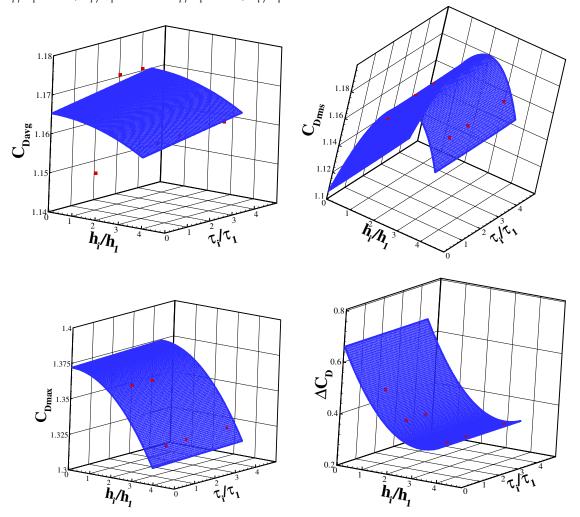


Figure 1 - Flow around a smooth cylinder at  $Re_D = 10^5$  computed with the SST k- $\omega$  turbulence model. Fits performed to the average ( $C_{Davg}$ ), root mean squared ( $C_{Drms}$ ) and maximum ( $C_{Dmax}$ ) values of the drag coefficient and to the amplitude of oscillation ( $\Delta C_D = C_{Dmax} - C_{Dmin}$ ). Submitted data: red dots. Data fit: blue lines.

The results present a good example of the difficulties to control and assess numerical uncertainties in this type of calculations. All quantities plotted in figure 1 were

derived from the time history of the same flow quantity. However, the behaviour of the convergence with grid and/or time step refinement is significantly different:

- The average and maximum values of the drag coefficient exhibit monotonic convergence, whereas  $C_{Drms}$  and  $\Delta C_{D}$  present non-monotonic convergence.
- $C_{Davg}$  and  $C_{Dmax}$  exhibit second-order convergence in space and first-order convergence in time. However, there is a significant amount of scatter in the  $C_{Davg}$  data (the standard deviation of the fit is of the order of the difference between the data), whereas  $C_{Dmax}$  presents a negligible value of the standard deviation of the fit.
- The two cases with non-monotonic convergence,  $C_{Drms}$  and  $\Delta C_D$ , do not show the same trend for the space and time discretization. For the space refinement, the convergence is not monotonic, whereas the convergence with the time step (lines with constant  $h_i/h_1$ ) shows monotonic convergence.

A second example of the error estimation performed to obtain the numerical uncertainty of the submitted results is presented in figure 2. The test case is again the flow around a smooth cylinder, but in this case the data is the average value of the drag coefficient ( $C_{Davg}$ ) obtained by the same code using the same turbulence model (k- $\omega$  SST [3]) at four different Reynolds numbers ( $Re_D=10^3$ ,  $Re_D=10^4$ ,  $Re_D=10^5$  and  $Re_D=5\times10^5$ ). The number of data points and the grid and time step refinement ratios depend on the selected Reynolds number. Nevertheless, it is clear that the convergence of  $C_{Davg}$  is dependent on the selected Reynolds number. There is one case with non-monotonic convergence in space and time ( $Re_D=10^3$ ) and another with monotonic convergence in space and time ( $Re_D=10^4$ ). For the two highest Reynolds numbers ( $Re_D=10^5$  and  $Re_D=5\times10^5$ ) the convergence is monotonic in time and non-monotonic in space.

Many more examples could be presented from the data submitted to the Workshop. The intention of this report is not to make an exhaustive demonstration of the difficulties to assess and control the numerical uncertainty of complex turbulent flow calculations. However, we want to emphasize the difference between the comparison of "single grid/time step calculations" and the comparison of numerical solutions with their respective numerical uncertainties. Therefore, in all the comparisons of submitted data we will include the estimated numerical uncertainty. Furthermore, the estimated uncertainty is one of the quantities required for the application of the V&V 20 ASME Validation procedure [2].

We must point out that non-monotonic convergence and/or a significant amount of scatter in the data (as illustrated in figures 1 and 2) will lead inevitably to large

estimated uncertainties. However, the aim of the comparisons of the next section is not to discuss the uncertainty estimation procedure (the same procedure was applied to all submissions). Our goal is to check if submissions using the same flow settings and the same mathematical model lead to overlapping error bars. If not, something is wrong in the numerical predictions (or the estimated uncertainties are under conservative).

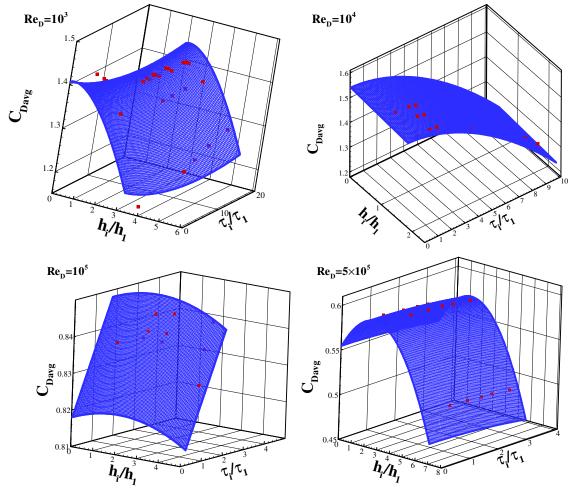


Figure 2 - Flow around a smooth cylinder at  $Re_D = 10^3$ ,  $Re_D = 10^4$ ,  $Re_D = 10^5$  and  $Re_D = 5 \times 10^5$  computed with the SST k- $\omega$  turbulence model. Fits performed to the average ( $C_{Davg}$ ) value of the drag coefficient. Submitted data: red dots. Data fit: blue lines.

# 5. Comparison of data

#### **5.1 Manufactured solutions**

As we mentioned above, the two groups that performed Code Verification using the Method of Manufactured Solutions (MMS) [6] have not followed exactly the proposed Manufactured Solution (MS). Therefore, we do not present any comparison of submitted data. Nevertheless, we would like to give one example of the capabilities of the MMS and its importance to assess the convergence properties of RANS solvers.

Figures 3 and 4 presents the grid convergence properties obtained for the horizontal velocity component  $(u_x)$  and for the eddy-viscosity  $(v_t)$  of the calculations of three two-dimensional, steady, incompressible manufactured solutions (MS1, MS2 and MS3) [7] performed with two RANS solvers using the one-equation Spalart & Allmaras model [5].

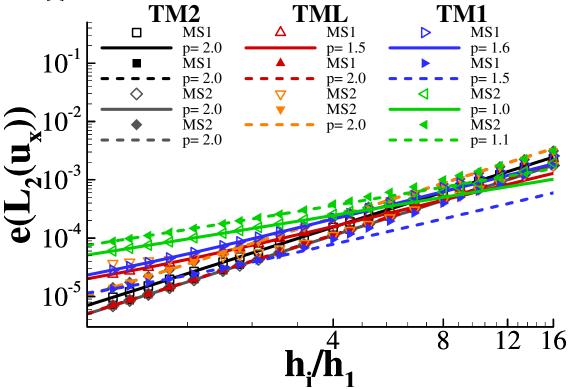


Figure 3 – Convergence of the  $L_2$  norm of the discretization error of the horizontal velocity component for three steady, two-dimensional, manufactured solutions (MS1, MS2 and MS3). Results obtained with the PARNASSOS and ReFRESCO RANS solvers using the one-equation model of Spalart & Allmaras. Different schemes in the discretization of the convective terms the turbulence model transport equation: TM2 – Second-order upwind without limiters; TML – Second-order upwind with limiters; TM1 – First-order upwind.

The plots included in figures 3 and 4 present the  $L_2$  norm of the discretization error as a function of the typical cell size in logarithmic scales. The two flow solvers use significantly different discretization techniques: finite-differences for non-orthogonal structured curvilinear grids (PARNASSOS) and faced based finite-volume for cells of arbitrary shape (ReFRESCO). The calculations were performed with at least second-order accurate discretization schemes for the continuity and momentum equations in sets of geometrically similar Cartesian grids. However, different schemes were tested in the discretization of the convective terms of the turbulence model transport equation: first-order upwind (TM1); second-order upwind with (TML) and without (TM2) limiters.

The convergence properties exhibit several interesting features:

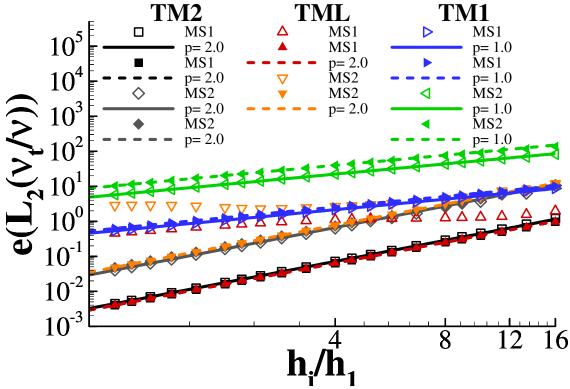


Figure 4 – Convergence of the  $L_2$  norm of the discretization error of the eddy-viscosity for three steady, two-dimensional, manufactured solutions (MS1, MS2 and MS3). Results obtained with the PARNASSOS and ReFRESCO RANS solvers using the one-equation model of Spalart & Allmaras. Different schemes in the discretization of the convective terms the turbulence model transport equation: TM2 – Second-order upwind without limiters; TML – Second-order upwind with limiters; TML – First-order upwind.

- As expected, second-order grid convergence is obtained for the TM2 scheme in both codes.
- Naturally, a first-order scheme used in the turbulence model transport equations (TM1) makes the order of grid convergence of the eddy-viscosity  $(v_t)$  drop to 1 for both codes. However, the same effect is observed for the mean flow horizontal velocity component  $(u_x)$ .
- The effect of the limiter in the second-order convection scheme (TML) is much stronger in the finite-difference code (PARNASSOS) than in the finite-volume solver (ReFRESCO). In PARNASSOS, the discretization error is shifting from the TM2 to the TM1 solution with grid refinement.

Such type of convergence properties would be extremely hard to discuss in the context of practical complex flows.

#### 5.2 Circular cylinder

There were results submitted for five different Reynolds numbers (see table 2): (  $10^3$ ,  $10^4$ ,  $10^5$ ,  $5\times10^5$  and  $10^6$ ). In this section, we present the comparison between the different submissions including the estimated uncertainties. This means that submissions that did not include enough data points to estimate the numerical uncertainty will not be included.

Although the main goal of this test case is Solution Verification, i.e. assessing the quality of numerical solutions, we will include the available experimental data in the plots of the following sections. Unfortunately, we do not have the experimental uncertainty. Nevertheless, for some flow conditions there is more than one measurement.

# **5.2.1** Drag coefficient, $C_D$

Figure 5 presents the drag coefficient data. The requested data included the average  $(C_{Davg})$ , root mean squared  $(C_{Drms})$  and maximum  $(C_{Dmax})$  values of the drag coefficient and the amplitude of oscillation  $(\Delta C_D = C_{Dmax} - C_{Dmin})$ . Experimental data is only available for the average drag coefficient.

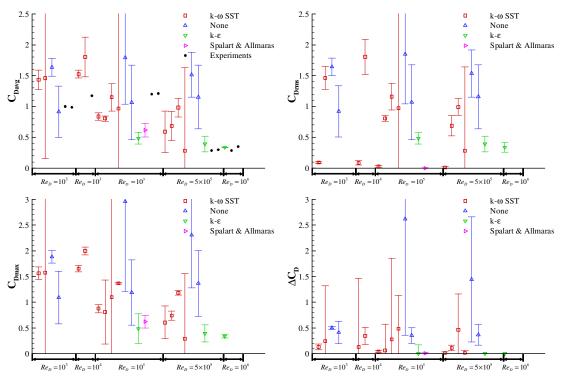


Figure 5 - Submitted data for the flow around a smooth cylinder. Average  $(C_{Davg})$ , root mean squared  $(C_{Drms})$  and maximum  $(C_{Dmax})$  values of the drag coefficient amplitude of oscillation  $(\Delta C_D = C_{Dmax} - C_{Dmin})$ .

Most of the estimated uncertainties are too large. As mentioned above, this is mostly a consequence of the significant amount of scatter in some of the submissions and/or non monotonic convergence in space and/or time for the grid refinement and time step levels used in the calculations. Nevertheless, there are several examples of calculations performed with the same turbulence model that do not exhibit overlapping error bars. Furthermore, predictions that show consistent error bars for one feature of the drag time history do not show it for others.

The comparison of the numerical predictions with the experimental  $C_{\text{Davg}}$  suffers from the absence of the experimental uncertainty. Nevertheless, as mentioned above, there are two experimental points for four of the five Reynolds numbers included in figure 8. Surprisingly, the discrepancies between the predictions and the experiments are largest for the two smallest Reynolds numbers.

# **5.2.2** Lift coefficient, $C_{I}$

The submitted data for the lift coefficient  $(C_L)$  is presented in figure 6. In this case, there is experimental data for  $C_{Lrms}$  at two of the selected Reynolds numbers and the expected average value should be zero. Some of the submissions did not report the average value of  $C_L$  and so the plot with  $C_{Lavg}$  contains less entries than the other three plots.

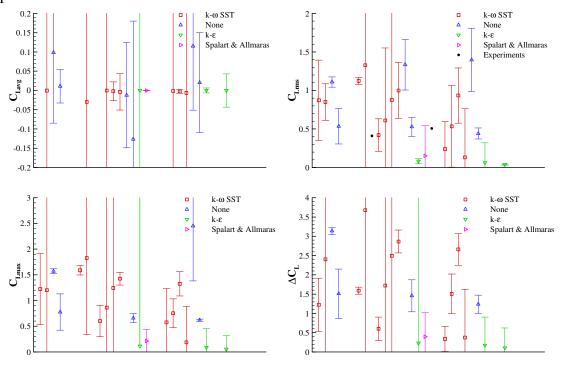


Figure 6 - Submitted data for the flow around a smooth cylinder. Average  $(C_{Lavg})$ , root mean squared  $(C_{Lrms})$  and maximum  $(C_{Lmax})$  values of the lift coefficient amplitude of oscillation  $(\Delta C_L = C_{Lmax} - C_{Lmin})$ .

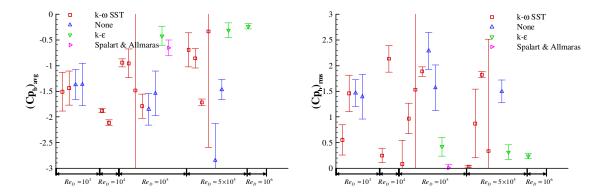


Figure 7 - Submitted data for the flow around a smooth cylinder. Average  $(Cp_b)_{avg}$  and root mean squared  $(Cp_b)_{rms}$  values of the base pressure coefficient.

In general, the estimated uncertainties are even higher than for the drag coefficient and the trends are similar to those observed in the drag coefficient data.

### 5.2.3 Base pressure coefficient, $Cp_h$

The results submitted for the base pressure coefficient  $(Cp_b)$  included some awkward numbers there are most likely due to a misinterpretation of the instructions. Therefore, figure 7 presents only the average  $(Cp_b)_{avg}$  and root mean squared  $(Cp_b)_{rms}$  values.

Although there are a few submissions with an "acceptable" estimated uncertainty, the range of values obtained for the base pressure coefficient is again impressive. Therefore, the results reinforce the main trends observed in the drag and lift coefficients: the difficulty to control (and assess) the numerical uncertainty of the predictions and the existence of inconsistencies in predictions performed with the same mathematical model.

# 5.2.4 Location of the separation point, $\theta_{sep}$

Not all the participants reported the values of the location of the flow separation point. Furthermore, in some cases different techniques have been used to determine its location. Therefore, the data available is not as complete as for the previous flow quantities and so we do not present any results in this comparison. Nevertheless, we must emphasize that the trends observed in the data are similar to those discussed above for the other flow quantities.

#### 5.2.5 Strouhal number, St

The last selected flow quantity is the Strouhal number St which was supposed to be determined from the first harmonic of the lift coefficient time history. Although there

are a few cases with too large estimated uncertainties, the Strouhal number is the variable with the most consistent set of results.

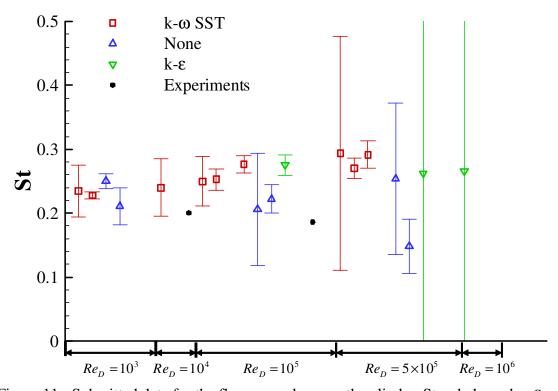


Figure 11 - Submitted data for the flow around a smooth cylinder. Strouhal number St.

#### 5.3 Straked riser

Only three groups submitted results for the straked riser test case and in some cases only preliminary calculations were performed up to the date of the Workshop. Therefore, we will restrict the comparison of the results to table 3 that contains the average and root mean squared drag coefficient,  $C_{\text{Davg}}$  and  $C_{\text{Drms}}$ , the root mean squared of the lift coefficient,  $C_{\text{Lrms}}$ , and the Strouhal number, St. It should be mentioned that submissions I and Ia are statistically steady flow calculations.

The table includes two submissions with the same code (I and Ia), which differ only in the size of the computational domain. One was performed with the recommended dimensions of the computational domain and the other with an inlet boundary moved upstream. These preliminary results suggest that the recommended size of the computational domain (location of the inlet boundary) has to be checked.

Naturally, the present submissions are not sufficient to obtain a reliable estimate of the numerical uncertainty and so we cannot apply the V&V20 ASME Validation procedure [2]. Nevertheless, the present submissions already lead to interesting discussions and suggestions for future improvement.

Submissions	I	Ia	II	III
$C_{\it Davg}$	1.659	1.89	1.242	1.867
$C_{\it Drms}$	1.659	1.89	1.242	1.867
$C_{\mathit{Lrms}}$	0.012	0.015	0.141	0.069
St			0.154	

Table 3 – Submitted results of the flow around a straked riser. Average and root mean squared drag coefficient,  $C_{\text{Davg}}$  and  $C_{\text{Drms}}$ , root mean squared lift coefficient  $C_{\text{Lrms}}$  and Strouhal number St.

#### 6. Final remarks

This first edition of the Workshop on Verification and Validation of CFD for Workshop flows showed the difficulties and advantages in assessing separately numerical (Verification) and modeling (Validation) errors in complex turbulent flows. This report presents a brief overview of the data submitted to this first edition. It represents a starting point for what we believe to be the most promising way to improve the accuracy of our numerical solutions, understand the limits of our present turbulence models and improve the quality of the mathematical models behind our CFD simulations.

We must emphasize that all sessions of the Workshop lead to very interesting and fruitful discussions. Therefore, we would like to thank to all the groups that submitted results to the Workshop (including those that have not attended the sessions) and to all the participants of the OMAE 2012 Conference that attended the sessions of this Workshop.

Since we believe that this type of events promotes the quality of CFD applications in the Offshore Industry, a second edition of this Workshop is proposed for the forthcoming OMAE 2013 Conference that will be held in June 2013 in Nantes. The same three test cases will be retained for the next edition of the Workshop including Code Verification, Solution Verification and Validation exercises. However, there are some small changes in the proposed exercises: the flow around the circular cylinder will be divided in two different exercises: Solution Verification for Reynolds numbers away from drag crisis; Validation for Reynolds numbers in the region of drag crisis. For the

flow around the straked riser it is recommended to the participants to check the influence of the inlet boundary on their predictions.

#### References

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