2^{nd} Workshop on CFD Uncertainty Analysis

FORTRAN Functions of Alternative Manufactured Solutions for One-equation Turbulence Models

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1 Manufactured Solution

1.1 General

This document presents the FORTRAN functions of an alternative manufactured solutions for the one-equation turbulence models. Most of the features of these MS are identical to the ones proposed in [1]. However, in these alternative solutions the manufactured dependent variables of the one-equation turbulence models of Menter, [2], and Spalart & Allmaras, [3], $\tilde{\nu}$, has been changed, [4]. This leads to different functions for the dependent variables of the one-equation models and to its derivatives with respect to the horizontal, x, and vertical, y, directions.

There are two alternatives proposed in [4]:

- Solution MS2

$$\tilde{\nu} = \nu_{max} \eta_\nu^2 e^{1-\eta_\nu^2} \, . \tag{1}$$

- Solution MS1

$$\tilde{\nu} = \nu_{max} \sqrt{2} \eta_\nu e^{0.5-\eta_\nu^2} \, . \tag{2}$$

$$\eta_\nu = \frac{\sigma_\nu y}{x} \, , \tag{3}$$

$\sigma_\nu = 2.5\sigma$, $\sigma = 4$ and ν_{max} is $10^3 \nu$.

The original MS, [1], is designated by MS4

- Solution MS4

$$\tilde{\nu} = 0.25 \nu_{max} \eta_\nu^4 e^{2-\eta_\nu^2} \, . \tag{4}$$

As for the original MS, MS4, [5], all the functions have been written in FORTRAN 90 with double precision (REAL*8) variables. The structure of the functions is identical for the three MS. The input arguments of all the functions are the Cartesian coordinates x and y. The argument of the damping functions of the one-equation models is the dependent variable of the model, $\tilde{\nu}$.

1.2 Main flow variables

1.2.1 *u* velocity component

Name	Arguments	Output
UMS	x, y	Horizontal velocity component, u
DUDXMS	x, y	Derivative of u with respect to x, $\frac{\partial u}{\partial x}$
DUDYMS	x, y	Derivative of u with respect to y, $\frac{\partial u}{\partial y}$
DUDX2MS	x, y	Second derivative of u with respect to x, $\frac{\partial^2 u}{\partial x^2}$
DUDY2MS	x, y	Second derivative of u with respect to y, $\frac{\partial^2 u}{\partial y^2}$
DUDXYMS	x, y	Second-order cross-derivative of u, $\frac{\partial^2 u}{\partial x \partial y}$

1.2.2 *v* velocity component

Name	Arguments	Output
VMS	x, y	Vertical velocity component, v
DVDXMS	x, y	Derivative of v with respect to x, $\frac{\partial v}{\partial x}$
DVDYMS	x, y	Derivative of v with respect to y, $\frac{\partial v}{\partial y}$
DVDX2MS	x, y	Second derivative of v with respect to x, $\frac{\partial^2 v}{\partial x^2}$
DVDY2MS	x, y	Second derivative of v with respect to y, $\frac{\partial^2 v}{\partial y^2}$
DVDXYMS	x, y	Second-order cross-derivative of v, $\frac{\partial^2 v}{\partial x \partial y}$

1.2.3 Pressure, C_p

Name	Arguments	Output
PMS	x, y	Pressure coefficient, $C_p = \frac{p - p_{ref}}{\rho U_{ref}^2}$
DPDXMS	x, y	Derivative of C_p with respect to x, $\frac{\partial C_p}{\partial x}$
DPDYMS	x, y	Derivative of C_p with respect to y, $\frac{\partial C_p}{\partial y}$

1.2.4 Eddy-Viscosity, ν_t

- One-equation turbulence model
  - Spalart & Allmaras

Name	Arguments	Output
EDDYSAMS	x, y	Eddy-Viscosity, ν_t
DESADXMS	x, y	Derivative of ν_t with respect to x, $\frac{\partial \nu_t}{\partial x}$
DESADYMS	x, y	Derivative of ν_t with respect to y, $\frac{\partial \nu_t}{\partial y}$

  - Menter

Name	Arguments	Output
EDDYMTMS	x, y	Eddy-Viscosity, ν_t
DEMTDXMS	x, y	Derivative of ν_t with respect to x, $\frac{\partial \nu_t}{\partial x}$
DEMTDYMS	x, y	Derivative of ν_t with respect to y, $\frac{\partial \nu_t}{\partial y}$

1.2.5 Auxiliary variables

Name	Arg.	Output
VORTMS	x, y	Magnitude of Vorticity, $S_\Omega = \left\lvert \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} \right\rvert$
STRAINMS	x, y	Strain-rate, $\sqrt{S} = \sqrt{2\left(\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right) + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2}$

1.3 Source terms of the momentum equations

1.3.1 One-equation turbulence models

- Spalart & Allmaras

Name	Arguments	Output
SMXSAMS	x, y	Source function of the x momentum equation, f_x
SMYSAMS	x, y	Source function of the y momentum equation, f_y

- Menter

Name	Arguments	Output
SMXMTMS	x, y	Source function of the x momentum equation, f_x
SMYMTMS	x, y	Source function of the y momentum equation, f_y

1.4 One-equation Turbulence models

1.4.1 Spalart & Allmaras

Name	Arguments	Output
SSAMS	x, y	Source function of the $\tilde{\nu}$ transport equation, f_{spal}
EM1MS	x, y	Dependent variable of the turbulence model, $\tilde{\nu}$
DEM1DXMS	x, y	Derivative of $\tilde{\nu}$ with respect to x, $\frac{\partial \tilde{\nu}}{\partial x}$
DEM1DYMS	x, y	Derivative of $\tilde{\nu}$ with respect to y, $\frac{\partial \tilde{\nu}}{\partial y}$
DEM1DX2MS	x, y	Second derivative of $\tilde{\nu}$ with respect to x, $\frac{\partial^2 \tilde{\nu}}{\partial x^2}$
DEM1DY2MS	x, y	Second derivative of $\tilde{\nu}$ with respect to y, $\frac{\partial^2 \tilde{\nu}}{\partial y^2}$
FV1SAMS	$\tilde{\nu}$	Damping function of the model
DFV1SAMS	$\tilde{\nu}$	Derivative of the damping function with respect to $\tilde{\nu}$

1.4.2 Menter

Name	Arguments	Output
SMTMS	x, y	Source function of the $\tilde{\nu}_t$ transport equation, f_{mnt}
EM1MS	x, y	Dependent variable of the turbulence model, $\tilde{\nu}_t$
DEM1DXMS	x, y	Derivative of $\tilde{\nu}_t$ with respect to x, $\frac{\partial \tilde{\nu}_t}{\partial x}$
DEM1DYMS	x, y	Derivative of $\tilde{\nu}_t$ with respect to y, $\frac{\partial \tilde{\nu}_t}{\partial y}$
DEM1DX2MS	x, y	Second derivative of $\tilde{\nu}_t$ with respect to x, $\frac{\partial^2 \tilde{\nu}_t}{\partial x^2}$
DEM1DY2MS	x, y	Second derivative of $\tilde{\nu}_t$ with respect to y, $\frac{\partial^2 \tilde{\nu}_t}{\partial y^2}$
D2MTMS	$\tilde{\nu}_t$	Damping function of the model
DD2MTMS	$\tilde{\nu}_t$	Derivative of the damping function with respect to $\tilde{\nu}_t$

References

[1] Eça L., Hoekstra M., Hay A., Pelletier D. - *A Manufactured Solution for a Two-Dimensional Steady Wall-Bounded Incompressible Turbulent Flow* - IST Report D72-34, EPM Report EMP-RT-2005-08, November 2005.

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[3] Spalart P.R., Allmaras S.R. - *A One-Equations Turbulence Model for Aerodynamic Flows* - AIAA 30th Aerospace Sciences Meeting, Reno, January 1992.

[4] Eça L., Hoekstra M., Hay A., Pelletier D. - *Manufactured Solutions for One-equation Turbulence Models in a Two-Dimensional Steady Wall-Bounded Incompressible Turbulent Flow* - IST Report D72-36, EPM Report EMP-RT-2006-02, February 2006.

[5] Eça L., Hoekstra M., Hay A., Pelletier D. - *FORTRAN Functions for the Two Test Cases of the 2^{nd} Workshop on CFD Uncertainty Analysis* - IST, November 2005.