1. Test cases description

The steady, two-dimensional, incompressible flows selected for test cases of the 2nd Workshop on CFD Uncertainty Analysis are:

1. A manufactured solution resembling a near-wall turbulent flow.

2. The flow over a backward facing step.

This choice is a consequence of the conclusions of the first edition of the Workshop, [1,2], which suggested the use of a manufactured solution as the first test case. As a consequence, the first test case is also a Code Verification exercise. The second proposed flow is taken from the ERCOFTAC Classic Database (C-30), [3]. This flow geometry has already been adopted for the first edition of the Workshop. However, in the present case the grids are free, whereas three sets of single-block structured grids were provided for the first Workshop.

1.1 Manufactured solution

1.1.1 Geometry and flow conditions

The geometry is a simple square domain with $0.5L<X<L$ and $0<Y<L$ or $0.5 \leq x \leq 1$ and $0 \leq y \leq 0.5$, where L is the reference length and upper case symbols are used for dimensional quantities and lower case symbols for dimensionless quantities. x is the horizontal coordinate and y the vertical coordinate. u_x and u_y designate the horizontal and vertical Cartesian velocity components and the pressure is given by $p = P/\rho U_{ref}^2$.

Using U_{ref} as the reference velocity, the Reynolds number is $R_e = \frac{U_{ref} L}{\nu} = 10^6$.

1.1.2 Boundary conditions

In this test case the exact solution is available and so one can choose the most convenient boundary conditions. Nevertheless, the manufactured solution, [4,5] was constructed to resemble a near-wall turbulent flow and so the following boundary conditions are mandatory:

$$x = 0 \Rightarrow \begin{cases} u_x = (u_x)_{ms} \\ u_y = (u_y)_{ms} \end{cases}$$

$$y = 0 \Rightarrow \begin{cases} u_x = (u_x)_{ms} = 0 \\ u_y = (u_y)_{ms} = 0 \end{cases}$$

where the subscript $_{ms}$ designates the manufactured solution.

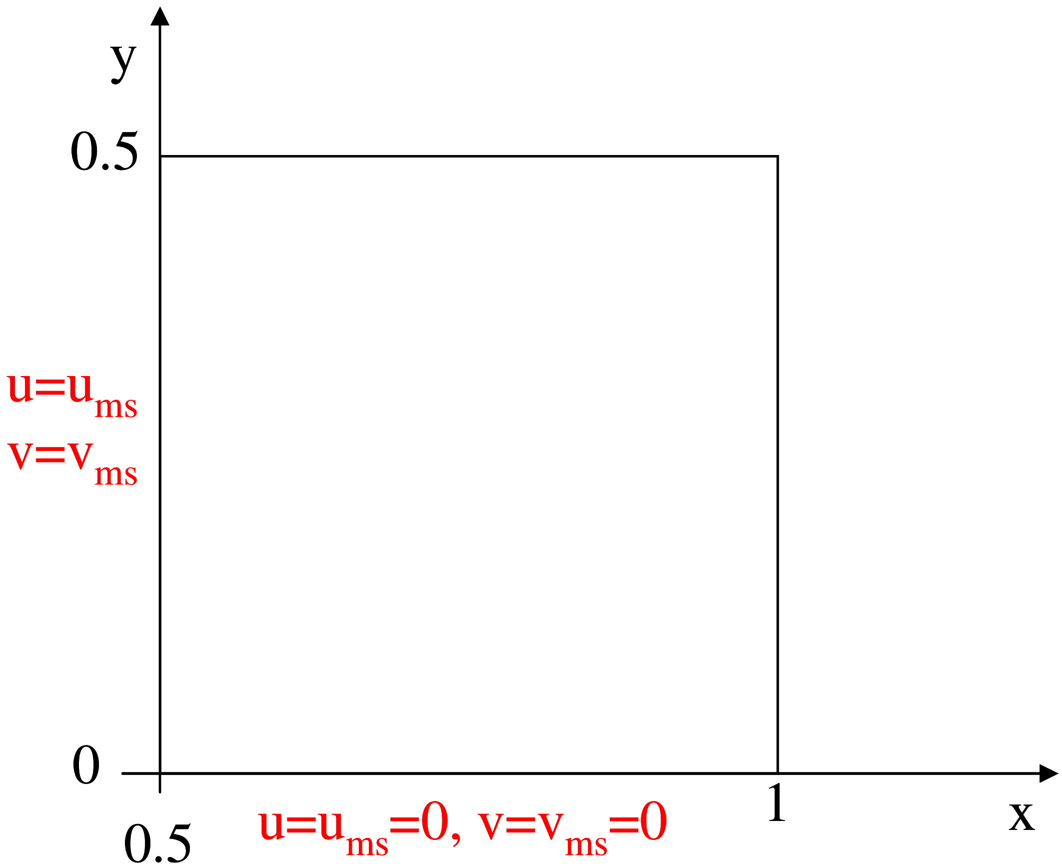

Figure 1 – Computational domain for the manufactured solution with the mandatory boundary conditions.

1.1.3 Exact solution

The exact solution uses the following the similarity variable

$$\eta = \frac{\sigma y}{x}$$

The proposed value of σ is 4.

1.1.4 Main flow variables

The main flow variables are given by

$$u_x = erf(\eta)$$

$$u_y = \frac{1}{\sigma\sqrt{\pi}}\left(1 - e^{-\eta^2}\right)$$

$$p = 0.5\ln(2x - x^2 + 0.25)\ln(4y^3 - 3y^2 + 1.25)$$

1.1.5 Turbulence quantities

The turbulence quantities depend on the turbulence model selected. There are manufactured solutions available for several eddy-viscosity one-equation and two-equation turbulence models, [4]:

- Spalart & Allmaras one-equation model, [6].
- BSL k-ω two-equation model proposed by Menter, [7].
- Menter one-equation model, [8].
- Standard k-ε two-equation model, [9].
- Chien's k-ε two-equation model, [10].
- TNT k-ω two-equation model, [11].

Two models have been recommended for the Workshop:

- Spalart & Allmaras one-equation model, [6].
- Baseline (BSL) k-ω two-equation model of Menter, [7].

1.1.5.1 Spalart & Allmaras model

The dependent variable of the Spalart & Allmaras one-equation turbulence model, $\tilde{\nu}$, is the eddy-viscosity, ν_t, multiplied by a damping function f_{v1}.

$$\nu_t = f_{v1}\tilde{\nu}$$

$$f_{v1} = \frac{\chi^3}{\chi^3 + 7.1^3}$$

$$\chi = \frac{\tilde{\nu}}{\nu}$$

The manufactured solution specifies $\tilde{\nu}$ and ν_t is obtained from the definition equation written above. There are two solutions available for $\tilde{\nu}$:

- MS4

$$\tilde{\nu} = 0.25\tilde{\nu}_{\max}\eta_\nu^4 e^{2-\eta_\nu^2}$$

- MS2

$$\tilde{\nu} = \tilde{\nu}_{\max}\eta_\nu^2 e^{1-\eta_\nu^2}$$

The suggested value of $\tilde{\nu}_{\max}$ is $10^3\nu$ and $\eta_\nu = \frac{\sigma_\nu y}{x}$ with $\sigma_\nu = 2.5\sigma = 10$. As discussed in [5], the MS4 solution may be troublesome to compute due to the near-wall behaviour of $\tilde{\nu}$.

1.1.5.2 BSL *k-ω* model

In the two equation BSL k-w model the manufactured quantities are the prescribed eddy-viscosity, ν_t, and the turbulence kinetic energy, k.

$$\nu_t = 0.25(\nu_t)_{\max}\eta_\nu^4 e^{2-\eta_\nu^2}$$
$$k = k_{\max}\eta_\nu^2 e^{1-\eta_\nu^2}$$

with k_{max}=0.01 and $(\nu_t)_{max}=10^3\nu$. ω is a consequence of the eddy-viscosity definition of the model.

$$\omega = \frac{k}{\nu_t} = 4\frac{k_{\max}}{(\nu_t)_{\max}}e^{-1}\eta_\nu^{-2}$$

In the BSL k-ω model, the constants of the production, dissipation and diffusion terms depend on the blending function F_1. As discussed in [12], the derivatives of the blending function are not uniquely defined in whole the computational domain due to its dependency on max-min functions. Therefore, for the present manufactured solution, the turbulence model can not be applied in its original form. In order to avoid problems due to the discontinuities of the blending function F_1, the constants of the diffusion terms are changed to $\sigma_k=\sigma_{k1}$ and $\sigma_\omega=\sigma_{\omega1}$.

1.1.6 Source Terms

The manufactured velocity field satisfies mass convergence, i.e. it is divergence free. However, the manufactured solutions do not satisfy the original form of the momentum equations and turbulence quantities transport equations. Therefore, balancing source terms must be added to these equations. Details on the source terms of the present manufactured solutions are given in [4,5] and Fortran 90 functions including all the required terms are available.

1.2 Flow over a backward facing step (Ercoftac C-30)

1.2.1 Geometry and flow conditions

The geometry of the experimental setup of this flow is illustrated in figure 2, which is taken from [3].

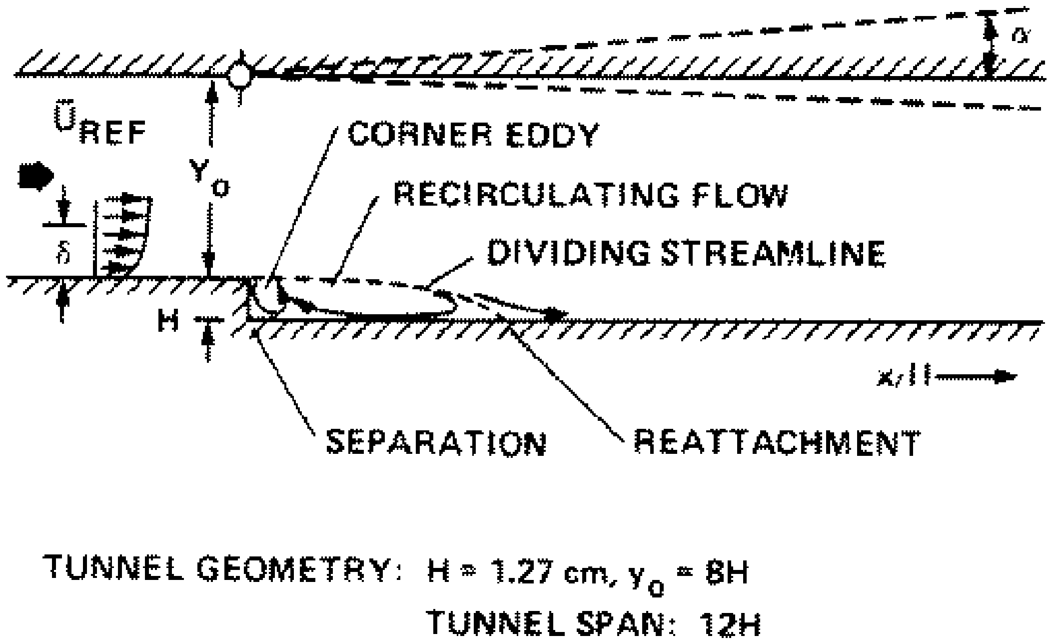

Figure 2 – Geometry of the flow over a backward facing step.

In the selected geometry, the angle of the top wall is 0 degrees. The velocity of the uniform incoming flow, U_{ref}, is 44.2 m/s and the step height, h, is 1.27 cm. The inlet is an *x=constant* section located 4 step heights upstream of the step and the outlet is an *x=constant* section 40 step heights downstream of the step. The Reynolds number based on U_{ref} and h is $R_e=50000$.

In the present edition of the Workshop, the grids are free.

1.2.2 Boundary conditions

The mandatory boundary conditions for the calculation of the flow over a backward facing step are:

- Velocity components on the bottom and top walls.
- Velocity components and turbulence quantities at the inlet.
- Pressure at the outlet.

The specification of the velocity components at the bottom and top walls is trivial because the impermeability and no-slip condition lead to zero velocity components. The pressure is also assumed to be constant at the outlet of the computational domain. At the inlet, the required flow quantities are specified with the multi-layer profiles adopted in the first edition of the Workshop, [1].

1.2.3 Inlet profiles

1.2.3.1 Velocity components

The Cartesian velocity component in the x direction, u_x, is defined with the help of analytical profiles. The present options were tuned to obtain a good agreement with the experimental results. The u_x profile in the vicinity of the two walls is assumed to be identical and so one only needs to specified it for half the distance between the two walls.

The vertical Cartesian velocity component in the y direction, u_y, is assumed to be zero.

In the experimental setup of this flow there is a uniform flow at the inlet with boundary-layer type profiles close to the two walls. The inlet conditions are given in [13] four step-heights upstream of the step, which has a height, h, of 1.27cm. At this location, the boundary-layer thickness, δ, is 1.9cm and the Reynolds number based on the inlet velocity, U_{ref}, and on the momentum thickness, θ, is

$$\mathrm{Re}_\theta = \frac{U_{ref}\,\theta}{\nu} = 5000.$$

The boundary-layer region is represented with a multi-layer profile using wall-coordinates, y^+ and u_τ.

$$y^+ = \frac{u_\tau y}{\nu} \text{ and } u_\tau = \sqrt{\frac{\tau_w}{\rho}} \text{ where } \tau_w \text{ is wall shear-stress.}$$

The u_x profile is specified with a three layer approach.

- For $y^+<25$:

A standard boundary-layer profile described in [14], which is defined from the momentum thickness, θ, and the skin friction coefficient, C_f. θ and C_f were selected to obtain the best agreement with the experimental data.

$$\frac{\theta}{h} = 0.15 \text{ and } C_f = 0.003.$$

With these choices of θ and C_f one obtains $\delta=1.99h$ at the inlet boundary.

- For $y^+\geq 25$ and $y<0.3\delta$:

$$\frac{u_x}{U_{ref}} = \left(\frac{y}{\delta}\right)^\gamma$$

where the exponent γ is obtained from the continuity of the u_x profile at $y^+=25$.

- For $y \geq 0.3\delta$:

The velocity profile is obtained with an Hermite interpolation. The derivative with respect to y at 0.3δ is obtained from the power-law profile and at δ is set equal to zero.

Figure 3 presents the inlet velocity profile and the experimental results, [13]. The standard boundary-layer profile suggested in [14] is also plotted in figure 3.

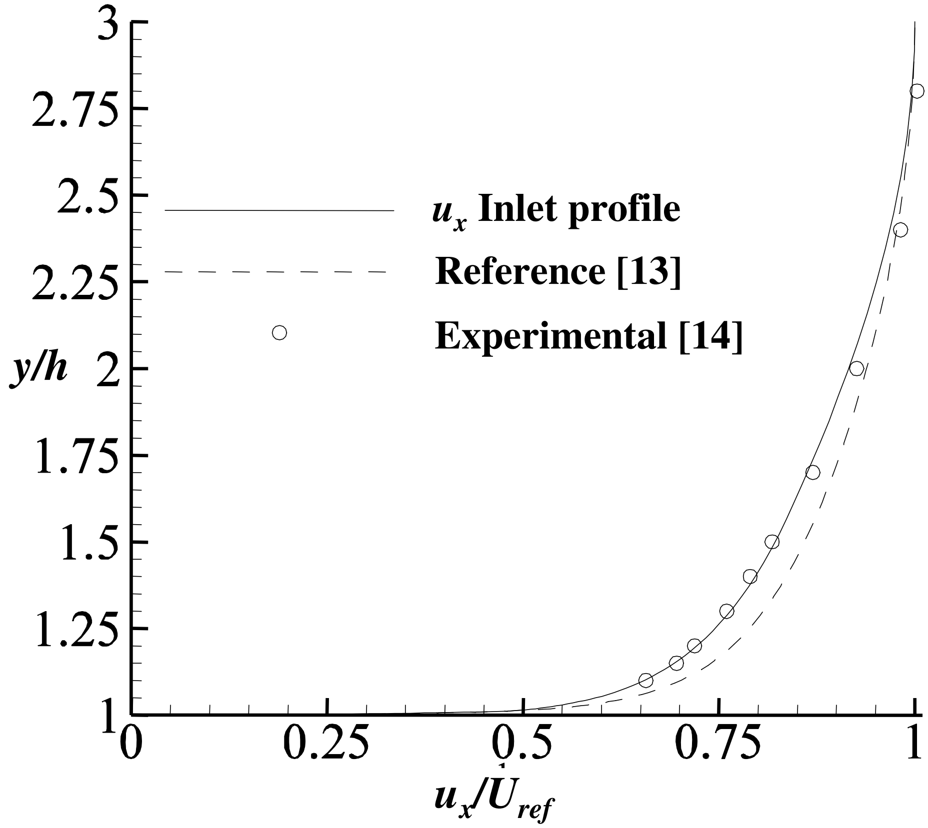

Figure 3 – Inlet u_x velocity profile for the steady, incompressible, 2-D flow over a backward facing step.

1.2.3.2 Turbulence quantities

The turbulence kinetic energy, k, has a constant value in the uniform flow region and a multi-layer profile in the near-wall region.

In the uniform flow region, the turbulent quantities are selected to obtain an eddy-viscosity equal to 0.01ν. Of the three turbulent quantities included in the two-equation models considered, k, ε and ω, the value of ω is known to be the most sensitive. Therefore, k and ε are specified with multi-layer profiles and ω is obtained as a consequence of the selected values of eddy-viscosity and k.

In the boundary-layer region the multi-layer profile is given by:

$$
\begin{aligned}
k^+ &= 0.05\left(y_n^+\right)^2 && \Leftarrow \quad y_n^+ < 5 \\
k^+ &= 1.25 + 0.325\left(y_n^+ - 5\right) && \Leftarrow \quad 5 \le y_n^+ < 15 \\
k^+ &= 4.5 - 3.6\eta^2 + 2.4\eta^3 && \Leftarrow \quad 15 \le y_n^+ < 60 \\
k^+ &= 3.3 && \Leftarrow \quad 60 \le y_n^+ \ \wedge \ y_n < 0.15\delta
\end{aligned}
$$

where $k^+ = \frac{k}{u_\tau^2}$, $\eta = \frac{y_n^+ - 15}{45}$ and y_n is the distance to the wall.

A cubic interpolation is applied between $y_n = 0.15\delta$ and the edge of the boundary layer using zero derivatives at $y_n = 0.15\delta$ and $y_n = \delta$.

For $y \ge \delta$,

$$k = \frac{0.1}{R_e} U_{ref}^2 \ .$$

ε is defined with different equations for the near-wall region and for the outer region of the boundary-layer.

For $y_n < 0.15\delta$, ε is obtained from

$$\varepsilon = \frac{k^{1.5}}{l}$$

with

$$l = 2.543687\, y_n \left(1 - e^{-R_k / 5.087374}\right)$$

and

$$R_k = \frac{\sqrt{k}\, l}{\nu} \ .$$

In the outer region ε is obtained with an Hermite interpolation for the region $0.15\delta < y_n < \delta$. The derivative at $y = \delta$ is set equal to zero and the derivative at 0.15δ is obtained from the linear variation between 0.15δ and δ.

For $y \ge \delta$,

$$\varepsilon = \frac{0.09}{R_e} \frac{U_{ref}^3}{h}$$

The eddy-viscosity, ν_t, profile is obtained using Chien's k-ε model, [10], with $\tilde{\varepsilon}$ given by

$$\tilde{\varepsilon} = \max\left(0,\, \varepsilon - \frac{2\nu k}{y_n^2}\right)$$

The dependent variable of the Spalart & Allmaras model, $\tilde{\nu}$, is calculated solving the non-linear equation that relates[1] ν_t to $\tilde{\nu}$

$$\nu_t = \frac{\tilde{\nu}^4}{\tilde{\nu}^3 + (7.1\nu)^3}\,.$$

The ω profile is specified with the help of the k and ν_t profiles with the exception of the near-wall viscous sub-layer.

$$\omega = \frac{6\nu}{0.075\, y_n^2} \qquad \Leftarrow \quad y_n^+ < 2.5$$

$$\omega = \frac{k}{\nu_t} \qquad \Leftarrow \quad 2.5 < y_n^+ \;\wedge\; y_n < \delta$$

$$\omega = 10\frac{U_{ref}}{h} \qquad \Leftarrow \quad \delta < y_n$$

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